

## Introduction

The process of cracks propagation is strongly intensified under the action of corrosion media and cyclic load. The long-term operation deteriorates the properties of the metal and, in particular, the fatigue crack growth resistance.

To determine the service life and the residual life, of metallic structural elements under cyclic loads and under the action of corrosive media, it is necessary to have the corresponding computational models.

Fundamental investigations of short cracks were carried out by Ritchie and Miller. The whole period of the fatigue crack growth was split into three stages according to the crack size (micro structurally short, physically small, and long cracks).

## Methods

### Problem statement

Consider a plate with a short rectilinear crack of the initial length  $2l_0$ . The plate is uniformly stretched by distributed cyclical forces  $p$  perpendicular to the line of the crack.

The problem is to determine the number of loading cycles  $N = N^*$  for which the corrosion-fatigue crack grows to the critical size  $l = l^*$  and the plate destroys.

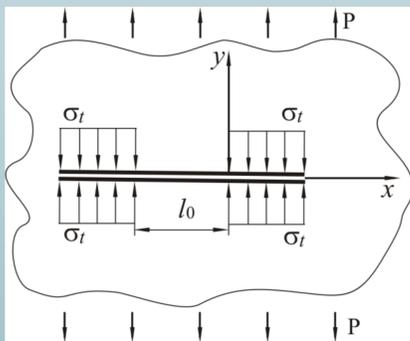


Fig. 1. Loading scheme of a cracked plate.

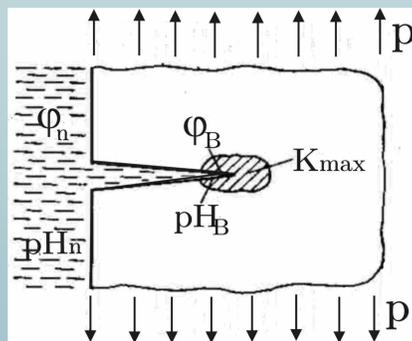


Fig. 2. Loaded plate with a rectilinear crack in a corrosive environment.

### Solution method and model development

The developed model is based on the first law of thermodynamics.

Power balance method:

$$A = W + \Gamma + Q + K, \quad (1)$$

where  $A$  – work of external forces,  $W$  – deformation energy of the material in the pre-fracture zone in the vicinity of the crack tip,  $\Gamma$  – fracture energy of the body.

The energy of deformation of the material in the pre-fracture zone in the vicinity of the crack tip is presented as:

$$W = W_S + W_p^{(1)}(l) + W_p^{(2)}(t) - W_p^{(3)}(t),$$

where  $W_S$  is the elastic component,  $W_p^{(1)}(l)$  – work of plastic deformations, which depends only on the crack length  $l$ ,  $W_p^{(2)}(t)$  – work of plastic deformation from external forces, which emanates at the constant crack length during the incubation period of its jump preparation,  $W_p^{(3)}(t)$  – work of plastic deformation during the unloading of the body and compression of the pre-fracture zone.

Change rate of the power balance:

$$\frac{\partial A}{\partial t} = \frac{\partial W}{\partial t} + \frac{\partial \Gamma}{\partial t}.$$

### Analytical model

with stress intensity factor (SIF)

$$\frac{dl}{dt} = \frac{\alpha(K_{I_{max}}^2 - K_{Scc}^2)[(1-R)^4(K_{I_{max}}^2 + K_{Scc}^2) + \eta E \sigma_t]}{E \sigma_t (K_{fcc}^2 - K_{I_{max}}^2)}, \quad (2)$$

$$N = 0, l(0) = l_0; \quad N = N_*, l(N_*) = l_*; \quad K_{I_{max}}(l_*) = K_{fcc}$$

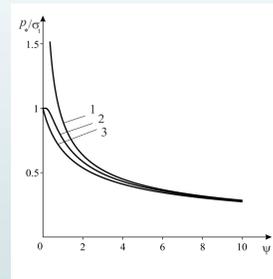
new model

$$\frac{dl}{dt} = \frac{\alpha(\delta_{tmax} - \delta_{scc})[(1-R)^4(\delta_{tmax} + \delta_{scc}) + \eta]}{\delta_{fcc} - \delta_{tmax}}, \quad (3)$$

$$N = 0, l(0) = l_0; \quad N = N_*, l(N_*) = l_*; \quad \delta_{tmax}(l_*) = \delta_{fcc}$$

## Results

### Disclosure determination in the top of a short crack under tension



$$\delta_t \approx K_I^2 [E \sigma_t (1 - \xi^2)]^{-1} \quad (4)$$

Fig. 3. Solutions comparison of the Griffiths task:

1 – via the Irwin criterion;

2 – via the  $\delta c$ -model;

3 – solution (1)

### Validation of the approximate formula to determine $\delta$

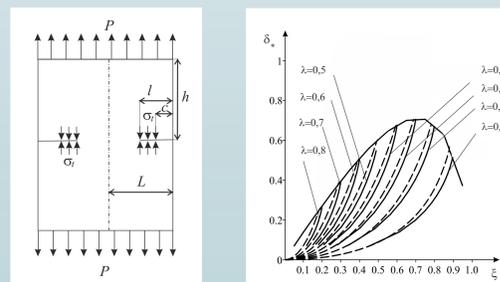


Fig. 4. Tension diagram of a plate with two side cracks (a); the dependence of the relative disclosure of the top of the cracks: solid lines – numerical data of Hayes and Williams, dotted line – dependence according to the formula (6) (b)

### Determination of the short fatigue crack propagation rate in plates

$$V = dl/dt = \alpha(1-R)^4 (\delta_{tmax}^2 - \delta_{th}^2) (\delta_{fcc} - \delta_{tmax})^{-1}$$

$$V = dl/dt = \alpha(1-R)^4 (K_{I_{max}}^4 - K_{th}^4) [E \sigma_t (K_{fcc}^2 - K_{I_{max}}^2)]^{-1}$$

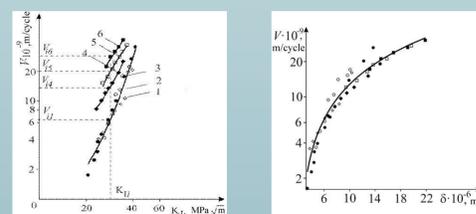


Fig. 5. Kinetic diagrams a)  $V \sim K_I$  b)  $V \sim \delta$  propagation of corrosion fatigue cracks in alloy Fe-3%, Si under load levels: 1- 560 MPa; 2 – 640 MPa; 3 – 720 MPa; 4 – 800 MPa; 5 – 840 MPa; 6 – 880 MPa

### Propagations of corrosion fatigue cracks numerical experiment

$$\alpha \approx 0,3(\text{cycle})^{-1}, \eta \approx 10^{-5} \text{MPa} \cdot \text{m}, \delta_{scc} \approx 2 \cdot 10^{-4} \text{mm},$$

$$\delta_{fcc} \approx 0,08 \text{mm}, R = 0,1, E = 2 \cdot 10^5 \text{MPa}, \sigma_t = 636 \text{MPa},$$

$$K_{fcc} \approx 101 \text{MPa} \sqrt{\text{m}}, K_{icc} \approx 9 \text{MPa} \sqrt{\text{m}}$$

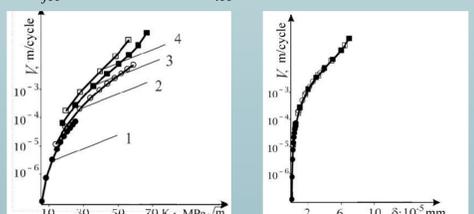


Fig. 6. Kinetic diagrams a)  $V \sim K_I$ , b)  $V \sim \delta_t$  propagation of corrosion fatigue cracks for series of load changes in the numerical experiment: 4 – 500 MPa, 3 – 450 MPa, 2 – 350 MPa, 1 – 150 MPa

### Evaluation of periods subcritical growth of short corrosion-fatigue cracks

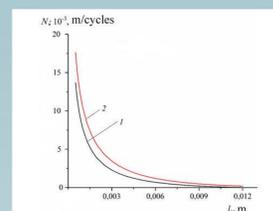


Fig. 7. Dependence of periods subcritical growth of short corrosion-fatigue cracks

$$N_* = E \sigma_t \alpha^{-1} \int_{l_0}^{l_*} \frac{(K_{fcc}^2 - K_{I_{max}}^2) dl}{(K_{I_{max}}^2 - K_{scc}^2) [(1-R)^4 (K_{I_{max}}^2 + K_{scc}^2) + \eta E \sigma_t]}$$

$$N_* = E \sigma_t \alpha^{-1} \int_{l_0}^{l_*} \frac{(1 - \xi^2) (K_{fcc}^2 - K_{I_{max}}^2) dl}{(K_{I_{max}}^2 - K_{scc}^2) [(1-R)^4 (K_{I_{max}}^2 + K_{scc}^2) + \eta E \sigma_t (1 - \xi^2)]}$$

## Conclusions

1. A general power balance method is proposed to investigate the propagation of short fatigue crack growth in plates under the action of loading and physicochemical factors. The approach is based on the first law of thermodynamics (1) and on a simplified formula (2) for determination of crack tip opening displacement using SIF and load level of the fracture process zone.
2. Having compared the experimental data, computational models founded on SIF (2) and the approach (3) (Fig. 5,6), it was shown that SIF model describes kinetics of the short crack growth no uniquely and may lead to big inaccuracies (Fig. 7) while determining their subcritical growth.

## Bibliography

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