Learning spaces: algorithms and structure

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Introduction

- With new technologies come new ways of learning and teaching. The theory of knowledge spaces [3] aims to benefits from computers power to improve or propose a new way to assess knowledge of students based on a mathematical structure named learning space. It has been implemented in the ALEKS system in USA, and is studied within the ProFan project in France, to which the PhD is attached. First, we explain learning spaces and thesis objectives. Then, we stick to one problem for which we state main ideas of existing results and current work.

Learning spaces with hands

Our main context: a group of students must master a topic at school.

- For each student, we want to reveal both her/his knowledge on the subject and what he/she is ready to learn.
- The subject is divided in a group of small problems called items. The set of items mastered by a student is its knowledge state. From this state we know which items to teach next.
- Assumptions: a student can learn one item at a time (see augmentation), the union of two (knowledge) states is a state too (see stability). The collection of possible states is called a learning space.
- We can discover a learning space by asking queries to expert teachers (see implications) of the form *"if a student fails this item, will she/he fails this one too?"*.

PhD Objectives

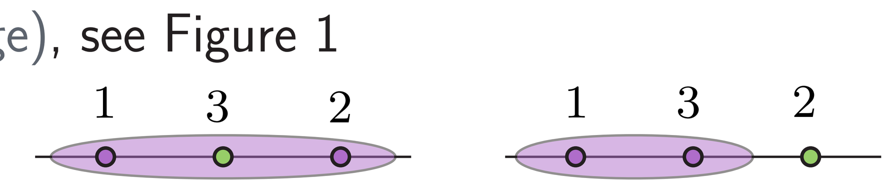
1. representation: how do we store efficiently the information of the structure ?
2. modification: how can we update the learning space ?

Both those questions first motivate the theoretical study of the mathematical structure hidden behind learning spaces. Second, results may lead to real applications (e.g: reducing memory costs, computation times).

Learning spaces in theory (a few)

- We give two possible definitions, see [4, 3]. They settle the more used dual but equivalent structure to learning spaces. Let Q be a finite set representing items, $F, G \subseteq Q$, $a, b \in Q$. A learning space is

(i) an anti-exchange closure space: a pair (Q, ϕ) where $\phi : 2^Q \mapsto 2^Q$ is an operator satisfying
  - ▷ $F \subseteq \phi(G) \iff \phi(F) \subseteq \phi(G)$, (closure)
  - ▷ $\forall a \neq b$ s.t $a, b \notin \phi(F)$, it holds $a \in \phi(F \cup \{b\}) \implies b \notin \phi(F \cup \{a\})$ (anti-exchange), see Figure 1

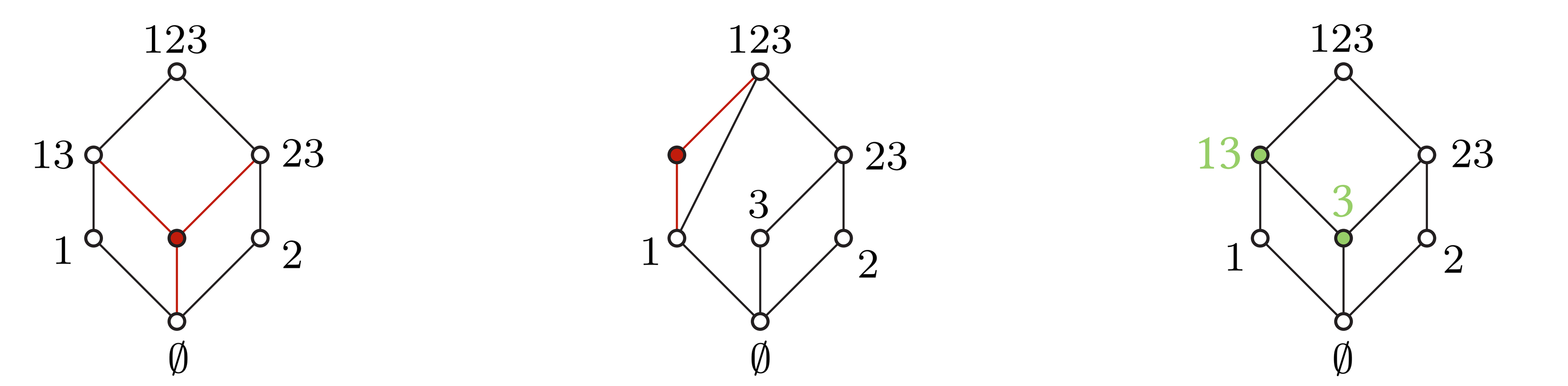


(a) focus on $[1, 2]$ (b) focus on $[1, 3]$

Figure 1:On this line $\mathbf{3} \in [\mathbf{1}, \mathbf{2}]$ (a) but $\mathbf{2} \notin [\mathbf{1}, \mathbf{3}]$ (b): they cannot be exchanged.

(ii) a convex geometry (see Figure 2) which is a pair $(Q, \mathcal{F})$, where $\mathcal{F} \subseteq 2^Q$ $(Q \in \mathcal{F})$ and
  - ▷ $F, G \in \mathcal{F} \implies F \cap G \in \mathcal{F}$, ($\cap$-stability)
  - ▷ for $F \subset Q$, $F \in \mathcal{F}$, $\exists q \in Q \setminus F$ s.t $F \cup \{q\} \in \mathcal{F}$. (augmentation)

- Under inclusion order, a $\cap$-stable set family is a lattice.
- There is a one-to-one correspondence between (Q, ϕ) and $(Q, \mathcal{F})$.
- F is closed or convex if $F = \phi(F)$ (equivalently $F \in \mathcal{F}$).



(a) failing $\cap$-stability (b) failing augmentation (c) valid convex geometry

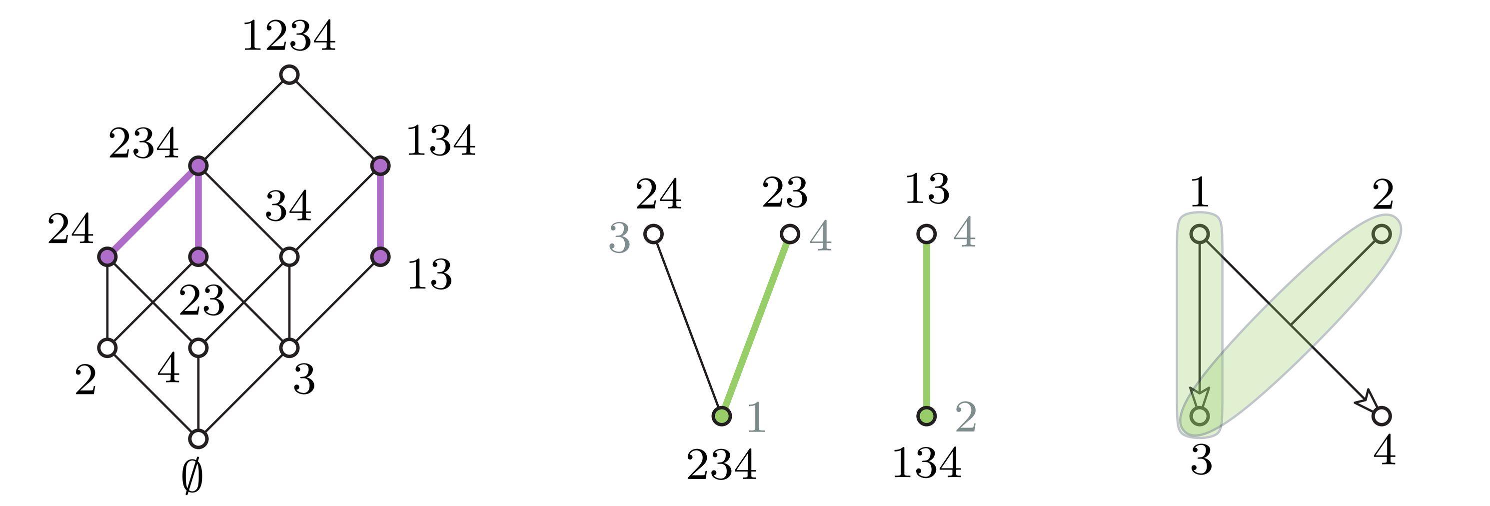
Figure 2:Illustration of convex geometry properties on $Q = \{\mathbf{1}, \mathbf{2}, \mathbf{3}\}$ through lattices of closed sets: in (a) $\mathbf{13} \cap \mathbf{23}$ is missing, in (b) $\mathbf{1}$ cannot be augmented. (c) is the geometry associated to convex sets of the line in Figure 1 (sets are ordered by inclusion and a sequence such as $\mathbf{123}$ is a shortcut for $\{\mathbf{1}, \mathbf{2}, \mathbf{3}\}$).

Ongoing topic: representation problem

Focus on the translation between representations. $(Q, \mathcal{F})$ is a learning space.

- Two widely used representations ([1]):
  - ▷ a characteristic subset $\mathcal{M}$ of $\mathcal{F}$ from which we can reconstruct $\mathcal{F}$,
  - ▷ a set $\boldsymbol{\Sigma}$ of implications $A \to B$ meaning *"if a student fails A, he/she fails B too"* $(A, B \subseteq Q)$.
- Existing results:
  - ▷ $\mathcal{M}$ and $\boldsymbol{\Sigma}$ uniquely define a learning space [5],
  - ▷ translation from one to another has unknown complexity [5],
  - ▷ existing algorithms for subclasses of closure systems [1, 2].
- Our ideas: algorithms for a subclass of convex geometries with an acyclicity constraint. Main tools are lattices, maximum independent sets of an hypergraph.

Example of translation



(a) Convex geometry (b) Characteristic sets (c) Implications

Figure 3:The lattice of (a) can be both represented by the implications $\{\mathbf{12} \to \mathbf{4}, \mathbf{1} \to \mathbf{3}\}$ in (c) and by the characteristic sets of (b) ordered by $\supseteq$. These sets are also highlighted in (a) along with their relationship.

We want to switch between (b) and (c) in Figure 3. For instance with $\mathbf{4}$:

- From (b) to (c): minimum set covering $\mathbf{4}$ in (b) is $\{\mathbf{234}, \mathbf{134}\}$ associated to $\{\mathbf{1}, \mathbf{2}\}$ so we derive $\mathbf{12} \to \mathbf{4}$ in (c) (the covering is denoted by highlighted arcs).
- From (c) to (b): maximum sets (circled) not implying $\mathbf{4}$ in (c) are $\mathbf{13}$ and $\mathbf{23}$, they do not fire the implication $\mathbf{12} \to \mathbf{4}$. Hence, they appear in (b) and are attached to $\mathbf{4}$.

Conclusion

- Learning spaces model knowledge of a particular topic through particular assumptions. They help to assess skills of students and point out what they should learn next.
- This theory wishes to use new technologies to improve educational systems. As such, our research is dedicated to representation and modification in view of real life applications such as ALEKS in USA or within the ProFan project in France.
- Representation problem has unknown tractability and put the light on the study of subclasses of learning spaces.

References

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