LIMOS CNRS UMR 6158 Axis MAAD

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Title of PhD subject

Efficient Extended formulations and Learning for the Traveling Salesman Problem and its variants

Summary :

Context In combinatorial optimization, an *extended formulation* refers to a linear (or integer) formulation of a problem that introduces additional variables beyond those needed to describe feasible solutions directly. By leveraging extra variables, extended formulations can often be more compact than the natural formulation based on the convex hull of feasible solutions. Due to this compactness, extended formulations have been widely applied in practical combinatorial optimization problems, such as vehicle routing and facility location. However, for fundamental combinatorial optimization problems like the Matching and Traveling Salesman Problems, most previous research has focused on the theoretical aspects—primarily the complexity of extended formulations—rather than practical implementation. A notable exception is our recent work [3], where we developed a branch-and-cut algorithm using an extended formulation for the Max-Cut problem. This approach demonstrated improved efficiency over the natural formulation within the same branch-and-cut framework. The proposed PhD research aims to extend this approach to the Traveling Salesman Problem (TSP). Another aspect is the use of machine learning techniques for accelerating branch-and-cut framework for TSP. Our recent works on the subject [4], [5] where we propose a machine learning framework for generating efficiently the combinatorial cuts in branch-and-cut have open up many interesting avenues for further research, which could be explored in a new PhD project.

Goals and Roadmap Several extended formulations have been proposed for the TSP. Among these, the MTZ formulation [1] is well-known for using additional variables to represent the sequence of vertices in the tour. This produces a compact formulation with big-M constraints and auxiliary real variables that range from 2 to n (where n is the number of vertices). However, due to these added components, the MTZ formulation often performs worse than the non-compact DFJ formulation [2], which uses only edge-based variables.

In the first part of this PhD, the candidate will explore alternative extended formulations for the TSP that build upon, rather than replace, the DFJ formulation. Here, the additional variables may capture weaker properties than those in the MTZ model but will remain binary, with no need for extra big-M constraints. The candidate will implement a branch-and-cut framework incorporating these proposed extended formulations to solve the TSP and its variants. Numerical results are anticipated to show that the new formulation outperforms the DFJ formulation within the same branch-and-cut setup. An additional, more challenging aspect that could be explored in the PhD thesis is examining the 4/3-conjecture for the DFJ formulation through the lens of the proposed extended formulations. In the second part devoted to machine learning aspect, the PhD candidate focus to extend the framework built in [4],[5], in particular for other classes of combinatorial cuts built upon extended formulations proposed in the first part.

Situation The thesis will be conducted at the LIMOS Laboratory within the Combinatorial Optimization group, under the supervision of Mourad Baiou and Viet Hung Nguyen. A potential candidate is Huy Phuc Nguyen HA, a Master's student in the ICS program, who is currently working on the proposed PhD thesis topic for his Master's internship.

References

[1] C.E. Miller, A.W. Tucker, R.A. Zemlin. Integer programming formulations and traveling salesman problems. Journal of Association for Computing Machinery, 7 (1960), pp. 326-329
[2] G.B. Dantzig, D.R. Fulkerson, S.M. Johnson. Solution of a large-scale traveling salesman problem. Operations Research, 2 (1954), pp. 393-410
[3] V. H. Nguyen, M. Minoux. Linear size MIP formulation of Max-Cut: new properties, links with cycle inequalities and computational results. Optim. Lett. 15(4): 1041-1060 (2021)
[4] T.Q.T Vo, M. Baiou, V.H. Nguyen, P. Weng. Improving Subtour Elimination Constraint Generation in Branch-and-Cut Algorithms for the TSP with Machine Learning. LION 2023: 537-551
[5] T.Q.T Vo, M. Baiou, V.H. Nguyen, P. Weng. Learning to Cut Generation in Branch-and-Cut Algorithms for Combinatorial Optimization" to appear in ACM Transactionson Evolutionary Learning and Optimization.