LIMOS- MAAD-AGC

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Title of PhD subject: Algorithms and structure stable matchings

Summary :

The Stable Matching Problem (SMP) is a classic combinatorial optimization problem, which is extremely relevant to economists, computer scientists, mathematicians and game theory experts, ever since its introduction by [Gale and Shapley, 1962].

According to [Gusfield and Irving, 2002], the term matching implies that the individuals or elements of sets participating in the problem are to be matched or assigned to each other, based on some specified criterion. In this problem, the criterion is to achieve stability, considering the preferences fixed by the participants.

This general scenario provides a model for application of this problem in some particular cases. In the classical Stable Marriage Problem, the aim is to create a one-on-one matching between two sets of the same size, representing the heterosexual marriage between men and women. Another relevant application is the allocation of students to universities or residents to hospitals, ensuring that institutions do not admit more students/residents than the available places [Gusfield and Irving, 2002]. Each agent in the SMP has a particular choice function which determines their preferences concerning the agents of the second group. There are two fundamental properties related to the choice function theory that are sufficient for the existence of stable sets, as reported by [Roth, 1984]:

Substitutability: let an agent a available in the set of agents A be chosen. If a subset S is derived from A and a remains available in S, then a should be chosen from S.

Independence from rejected alternatives: if an agent a is not chosen from the set of agents A, then the choice from a set (A - a) remains the same as for A. In other words, the alternatives that are not chosen have no impact on the final choice.

The objective of this thesis is to consider the structural properties of the set of all stable matchings for general case, i.e. many to many. The idea is to understand how the behavior of the lattice structure and then develop a efficient algorithms to enumerate all many-to-many stable matchings.

References

1. Jugo Fuji, Substitutable choice functions and convex geometry, Discrete Applied Mathematics, Volume 186, 2015,
2. M. Aizerman and A. Malishevski. General theory of best variants choice: Some aspects. IEEE Transactions on Automatic Control, 26(5):1030–1040, 1981.
1. Hamed Hamze Bajgiran and Federico Echenique. Closure operators: Complex- ity and applications to classification and decision-making. In Proceedings of the 23rd ACM Conference on Economics and Computation, EC ’22, page 35–55, New York, NY, USA, 2022. Association for Computing Machinery.
2. Charles Blair. The lattice structure of the set of stable matchings with multiple partners. Math. Oper. Res., 13(4):619–628, 1988.
3. Tamas Fleiner. A fixed-point approach to stable matchings and some applica- tions. Math. Oper. Res., 28(1):103–126, 2003.
4. Dan Gusfield, Robert Irving, Paul Leather, and Michael Saks. Every finite dis- tributive lattice is a set of stable matchings for a small stable marriage instance. Journal of Combinatorial Theory, Series A, 44(2):304–309, 1987.
5. Yi-You Yang. Rationalizable choice functions. Games and Economic Behavior, 123:120–126, 2020.