MAINTAINING GRAPH DECOMPOSITIONS

PhD thesis at LIMOS, Clermont-Ferrand,
advisors : Mamadou M. Kanté and Bruno Guillon
Emails: {kante,guillon}@isima.fr.

Mots clés: logic; dynamic algorithm; Dyn-FO; MSO; database; graph

The goal is to study algorithmics on dynamic graphs, and in particular on graph databases.

During the nineties, Patnaik and Immerman introduced the notion of dynamic logic [11] to study the complexity of problems in dynamic graphs. A property is definable in Dyn-L with L a logical language, *e.g.*, FO or PROP, if one can firstly build a data-structure and maintain it with interpretations in L, and secondly be able to check, with an L-definable property in the data structure, whether the property is satisfied by the modified graph. For instance, bipartiteness, reachability, computation of a minimum spanning tree are all definable in Dyn-FO [7, 11], and none of them is /FO-definable.

An interesting line of research is to determine for a classical logical language L, which properties are definable in Dyn-L,, and lassical meta-theorems in descriptive complexity are natural candidates. Indeed, it is proved that regular word/tree languages are in Dyn-FO [9], and one can ask whether a similar theorem can be true for Courcelle's theorem [5]. Bouyer-Decitre *et al.* have shown that monadic second-order (MSOL) properties of bounded tree-width graphs are definable in Dyn-FO as long as the modifications do not alter a fixed tree decomposition [4].

Even though interesting, the result in [4] does not take into account all possible modifications. One can however, overcome this difficulty if only interested in showing that MSOL-definable properties in bounded tree-width graphs are definable in Dyn-FO [8]. But, the technique in [8] seems difficult to extend, while many meta-theorems are based on a specific decomposition [12].

In this thesis, we are interested in maintaining decompositions with nice properties when dealing with structured dynamic graphs, *i.e.*, at each time the current graph belongs to a well-structured graph class, for instance bounded tree/clique-width, nowhere dense, etc. [10, 6]. We would like to tackle not only decision problems, but also counting and listing ones. For an example, it is proved in [3] that one can define in MSOL a tree-decomposition of width k on any graph of tree-width k. The first objectives of the thesis are:

(1) Show that one can define in Dyn-FO a tree-decomposition of width $f(k)$ on dynamic graphs of tree-width k.
(2) Extend the results in [8] to counting and listing problems.

One can in a first step extend what was done in [13, Section 6] or look at special cases as done in [2] where the case of tree-width ≤ 2 dynamic graphs were considered.

In a second step, one would consider extend these results to more general graph classes such as the *nowhere dense* ones [10], and extend the meta-theorems on static graphs [12] to dynamic graphs. A first step might be a generalisation of [1] to dynamic graphs that have bounded local tree-width.

Applications. We are seeking for a student with a master degree in computer science or mathematics, and with graduate lectures in algorithms and/or logic.

References

[1] Christoph Berkholz, Jens Keppeler, and Nicole Schweikardt. Answering FO+MOD queries under updates on bounded degree databases. *ACM Trans. Database Syst.*, 43(2):7:1–7:32, 2018.

[2] Hans L. Bodlaender. Dynamic algorithms for graphs with treewidth 2. In *Graph-Theoretic Concepts in Computer Science, 19th International Workshop, WG '93, Utrecht, The Netherlands, June 16-18, 1993, Proceedings*, pages 112–124, 1993.

[3] Mikołaj Bojańczyk and Michal Pilipczuk. Definability equals recognizability for graphs of bounded treewidth. In *Proceedings of the 31st Annual ACM/IEEE Symposium on Logic in Computer Science, LICS '16, New York, NY, USA, July 5-8, 2016*, pages 407–416, 2016.

[4] Patricia Bouyer-Decitre, Vincent Jugé, and Nicolas Markey. Courcelle's theorem made dynamic. *CoRR*, abs/1702.05183, 2017.

[5] Bruno Courcelle. The monadic second-order logic of graphs. i. recognizable sets of finite graphs. *Inf. Comput.*, 85(1):12–75, 1990.

[6] Bruno Courcelle and Joost Engelfriet. *Graph Structure and Monadic Second-Order Logic: a Language Theoretic Approach*, volume 138 of *Encyclopedia of Mathematics and its Applications*. Cambridge University Press, 2012. Available at www.labri.fr/perso/courcell/Book/TheBook.pdf.

[7] Samir Datta, Raghav Kulkarni, Anish Mukherjee, Thomas Schwentick, and Thomas Zeume. Reachability is in dynfo. *J. ACM*, 65(5):33:1–33:24, 2018.

[8] Samir Datta, Anish Mukherjee, Thomas Schwentick, Nils Vortmeier, and Thomas Zeume. A strategy for dynamic programs: Start over and muddle through. *Logical Methods in Computer Science*, 15(2), 2019.

[9] Wouter Gelade, Marcel Marquardt, and Thomas Schwentick. The dynamic complexity of formal languages. *ACM Trans. Comput. Log.*, 13(3), 2012. Available at http://arxiv.org/abs/0812.1915.

[10] Jaroslav Nesetril and Patrice Ossona de Mendez. *Sparsity - Graphs, Structures, and Algorithms*, volume 28 of *Algorithms and combinatorics*. Springer, 2012.

[11] Sushant Patnaik and Neil Immerman. Dyn-fo: A parallel, dynamic complexity class. *Journal of Computer and System Sciences*, 55:199–209, 1997.

[12] Nicole Schweikardt, Luc Segoufin, and Alexandre Vigny. Enumeration for FO queries over nowhere dense graphs. In *Proceedings of the 37th ACM SIGMOD-SIGACT-SIGAI Symposium on Principles of Database Systems, Houston, TX, USA, June 10-15, 2018*, pages 151–163, 2018.

[13] Nils Vortmeier. *Dynamic Complexity under Complex Changes*. PhD thesis, Technischen Universität Dortmund, 2019. Available at https://eldorado.tu-dortmund.de/bitstream/2003/38515/1/Dissertation-Vortmeier.pdf.