MAINTAINING GRAPH DECOMPOSITIONS

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The goal is to study algorithmics on dynamic graphs, and in particular on graph databases.

During the nineties, Patnaik and Immerman introduced the notion of dynamic logic \cite{11} to study the complexity of problems in dynamic graphs. A property is definable in Dyn-L with L a logical language, \textit{e.g.}, FO or PROP, if one can firstly build a data-structure and maintain it with interpretations in L, and secondly be able to check, with an L-definable property in the data structure, whether the property is satisfied by the modified graph. For instance, bipartiteness, reachability, computation of a minimum spanning tree are all definable in Dyn-FO \cite{7,11}, and none of them is /FO-definable.

An interesting line of research is to determine for a classical logical language L, which properties are definable in Dyn-L, and classical meta-theorems in descriptive complexity are natural candidates. Indeed, it is proved that regular word/tree languages are in Dyn-FO \cite{9}, and one can ask whether a similar theorem can be true for Courcelle's theorem \cite{5}. Bouyer-Decitre \textit{et al.} have shown that monadic second-order (MSOL) properties of bounded tree-width graphs are definable in Dyn-FO as long as the modifications do not alter a fixed tree decomposition \cite{4}.

Even though interesting, the result in \cite{4} does not take into account all possible modifications. One can however, overcome this difficulty if only interested in showing that MSOL-definable properties in bounded tree-width graphs are definable in Dyn-FO \cite{8}. But, the technique in \cite{8} seems difficult to extend, while many meta-theorems are based on a specific decomposition \cite{12}.

In this thesis, we are interested in maintaining decompositions with nice properties when dealing with structured dynamic graphs, \textit{i.e.}, at each time the current graph belongs to a well-structured graph class, for instance bounded tree/clique-width, nowhere dense, etc. \cite{10,6}. We would like to tackle not only decision problems, but also counting and listing ones. For an example, it is proved in \cite{3} that one can define in MSOL a tree-decomposition of width $k$ on any graph of tree-width $k$. The first objectives of the thesis are:
(1) Show that one can define in Dyn-FO a tree-decomposition of width $f(k)$ on dynamic graphs of tree-width $k$.

(2) Extend the results in [8] to counting and listing problems.

One can in a first step extend what was done in [13, Section 6] or look at special cases as done in [2] where the case of tree-width $\leq 2$ dynamic graphs were considered.

In a second step, one would consider extend these results to more general graph classes such as the nowhere dense ones [10], and extend the meta-theorems on static graphs [12] to dynamic graphs. A first step might be a generalisation of [1] to dynamic graphs that have bounded local tree-width.

**Applications.** We are seeking for a student with a master degree in computer science or mathematics, and with graduate lectures in algorithms and/or logic.

**References**


