The inpainting technique using the EM algorithm

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Objectives

1. Restore a damaged area in visually plausible form using information outside the damaged domain.
2. Use the EM algorithm for to find the classes in each neighborhood (patch) and together with the color information to obtain a better description of the patches.

Introduction

- In the last years, different works have been present to cope with the problem of the reconstruction of missing data. One of the techniques used for the reconstruction of an image is the "inpainting" technique, whose objective is to restore a damaged area in visually plausible form using information outside the damaged domain. The proposed criterion for the search of the most similar patch includes the sum square error (SSE) both for the brightness of pixels as well as for the classes (modes) found in the image.

Problem Formulation

- Let us consider a image $I(x, y)$ with dimension of $M \times N$ pixels and partly damaged.
- I can be divided in two parts: a *target region* $\boldsymbol{\Omega}$, that represents the missing pixels and a *source region* $\boldsymbol{\Phi}$, from which the most similar patch is extracted for the reconstruction of $\boldsymbol{\Omega}$.

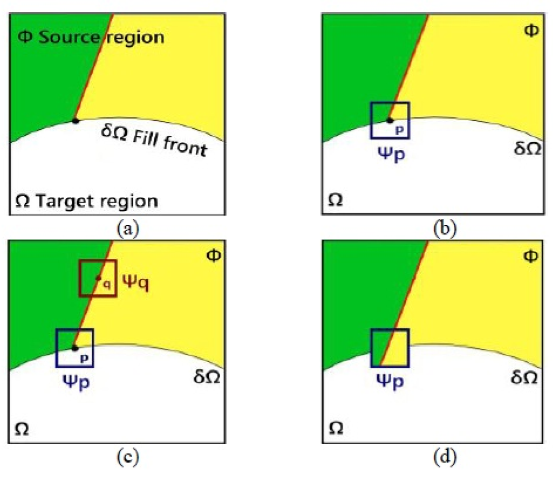

Figure 1: Inpainting

- To each pixel $p(x, y) \in \partial\Omega$ we assign an inpainting priority value $P(p)$ defined by:

$$P(p) = C(p).D(p) \tag{1}$$

where $C(p)$ and $D(p)$ represent the *confidence* and *data* terms respectively.

Methods

- To determine the classes within the image, we use the expectation-maximization (EM) algorithm.
  - Let $\tilde{X} = \{x(u, v) : x(u, v) \in I\}$ as incomplete data where the missing part is Z,
  - Suppose that L is the number of pixels in $\tilde{X}$, the missing part can be evaluated as a set of L labels $Z = \{z^{(1)}, z^{(2)}, \dots, z^{(L)}\}$.

$$z^{(i)} = [z_1^{(i)}, z_2^{(i)}, \dots, z_M^{(i)}]$$

    such that $z_r^{(i)} = 1$ $(r \in \{1, 2, \dots, M\})$ if the $i-$th pixel x^i of $\tilde{X}$ belongs to the $r-$th data ω_r, and $z_r^{(i)} = 0$ otherwise.
  - The complete log-likelihood function is given by

$$\ell(\Psi|\Theta) = \log p(\Psi|\Theta) = \sum_{i=1}^{L}\sum_{r=1}^{M} z_r^{(i)} \ln[P_r p(x^i|\theta_r)] \tag{2}$$

    where $\theta_r = [\mu_r, \sigma_r]$, and $P_r = P(\omega_r)$.
  - **E-step**: Compute $z_r^{(i)}$ given the parameter estimates from the previous M-step

$$z_r^{(i)} = \frac{P_r.N(x^i|\mu_r, \sigma_r)}{\sum_{j=1}^{M} P_j.N(x^i|\mu_j, \sigma_j)} \tag{3}$$

  - **M-step**: Obtain new parameter estimates (denoted by the prime)

$$P_r^{'} = \frac{1}{L}\sum_{i=1}^{L} z_r^{(i)} \tag{4}$$

$$\mu_r^{'} = \frac{\sum_{i=1}^{L} z_r^{(i)} x^i}{\sum_{i=1}^{L} z_r^{(i)}} \tag{5}$$

$$\sigma_r^{'} = \sqrt{\frac{\sum_{i=1}^{L} z_r^{(i)}(x^i - \mu_r^{'})}{\sum_{i=1}^{L} z_r^{(i)}}} \tag{6}$$

  - We can assign to each pixel x^i of $\tilde{X}$ the optimal class label:

$$\hat{\omega} = \arg\max_{\omega_r \in \Omega}\{P(\omega_r|x_i)\} \tag{7}$$

Mathematical Section

- The proposed criterion for the search of the most similar patch includes the sum square error (SSE):

$$\Psi_{\hat{q}} = \min_{\Psi_q \in \Phi} d(\Psi_{\hat{p}}, \Psi_q) \tag{8}$$

  where $d(\Psi_{\hat{p}}, \Psi_q) = SSE(\Psi_{\hat{p}}, \Psi_q) + SSE(class(\Psi_{\hat{p}}), class(\Psi_q))$
- The second criterion for the search is to use SSE for the classes (modes). We use equation (8) to find K similar patches.
  - In the patch $\Psi_{\hat{p}}$, we can distinguish two regions A of the known pixels and B of missing pixels. For each patch N_k, we make the same division of the area.
  - Given a pixel vector $x = [x_1, x_2, \dots, x_K]$, such that $x_j \in N_j$ represents the pixel value in the jth neighborhood. The linear predictor can be expressed as $y = f[x] = \sum_{j=1}^{K} \alpha_j.x_j$
  - To find the alpha values:

$$\begin{bmatrix} x_1^1 & x_2^1 & \dots & x_K^1 \\ x_1^2 & x_2^2 & \dots & x_K^2 \\ \vdots & \vdots & \ddots & \vdots \\ x_1^R & x_2^R & \dots & x_K^R \end{bmatrix} . \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \vdots \\ \alpha_K \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_R \end{bmatrix} \Leftrightarrow \underline{P}.\underline{\alpha} = \underline{Y} \tag{9}$$

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$$\alpha^* = (\underline{P}^t.\underline{P})^{(-1)}.\underline{P}^t.\underline{Y} \tag{10}$$

    The reconstruction phase of each missing data in $\Psi_{\hat{p}}$, consist in predict y by means of the linear predictor 9.

Results: Figure

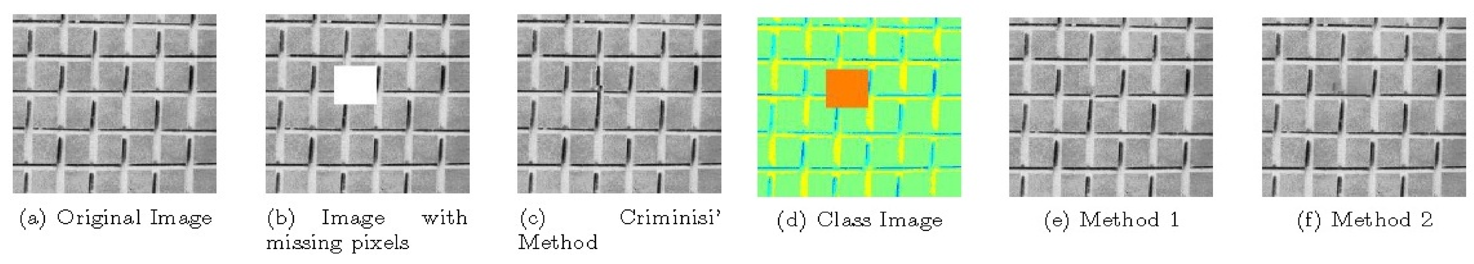

Figure 2: Test Image: Texture

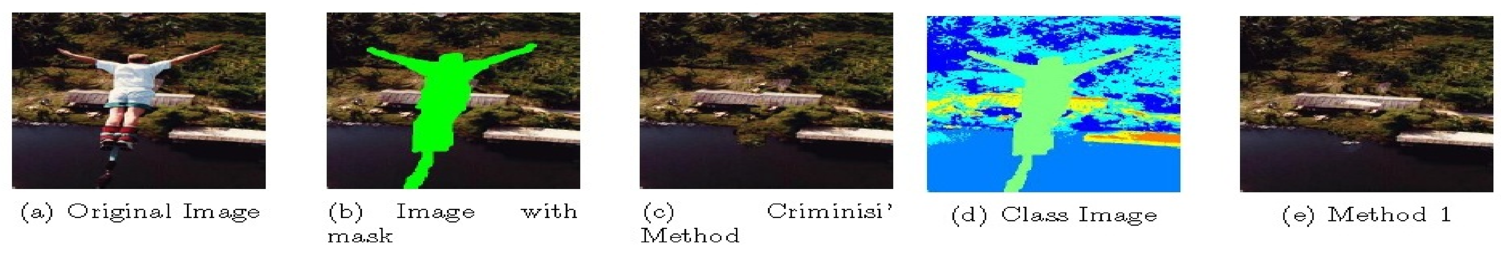

Figure 3: Test Image

Results: Table

- For quantify the reconstruction accuracy, we use the root-mean-square error (RMSE):

	Criminisi' Method	Method 1	Method 2
Texture 1	2.1180	1.7337	2.0889

Table 1: RMSE

Conclusion

- Combining the classes found in each neighborhood (patch) with the color information allows us to obtain a better description of the patches.
- Our experiments show a better reconstruction of the images, and they are visually almost equal to their original image.

References

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