Routing problems with inventory and loading constraints

Axel Delsol
LIMOS, UCA

Objectives

- Study complex vehicle routing problems with inventory and loading constraints.
- Propose exact approaches : Mixed Integer Programming (MIP), Constraint Programming (CP) ...
- Propose heuristic approaches : metaheuristics, hybrid methods ...

First studied problem

- Multivehicle Inventory Routing Problem = Capacitated Vehicle Routing Problem + Inventory management
- Handle inventory level over time.
- Schedule transportation of products.

Capacitated Vehicle Routing Problem (CVRP)

Data :
1. Complete weighted graph : G = (V, E)
2. Weight of edge (i, j) ∈ E : C_{i,j}
3. V = {0, 1, ..., N}
4. Number of nodes : N
5. Special node 0 : depot
6. Nodes i ≥ 1 : customers
7. Fleet of K vehicles with capacity Q
8. Demand at each node i ≥ 1 : R_i

Variables:
1. Vehicle assignment of node i ≥ 1 :
   \nu_i
2. Successor of node i ≥ 1 in its route : x_i
3. First visited node of vehicle k : x_0^k

Objective: Minimize
Transportation cost : ∑_{i≤N} C_{i,k} + ∑_{k∈K} C_{0,k}x_0^k

Inventory management (IM)

Data :
1. Discrete time horizon : T = {1, 2, ..., T}
2. Demand at each node i ≥ 1 at period t : q_i^t
3. Production at the supplier and period t : P_i^t
4. Inventory bounds at node i : [L_i, U_i]
5. Inventory cost of node i at period t : H_i^t
6. Initial inventory level at node t : I_0^t

Variables:
1. Inventory level of node i at period t : H_i^t
2. Delivery of node i ≥ 1 at period t : q_i^t

Objective: Minimize
Inventory cost : ∑_{i∈T} ∑_{t=0}^T H_i^t

Resolution scheme

Phase 1 : Feasible Inventory solution

Solved using CP (OR-Tools : https://developers.google.com/optimization/)
Inventory constraints:
- \( s_i^t = I_i^{t-1} - R_i^t + q_i^t \)
- \( s_i^t = I_i^{t-1} - R_i^t + q_i^t \)

Assignment constraints:
- \( ∀ t ∈ T \setminus \{1\}, s_i^t = s_i^{t-1} - R_i^t + q_i^t \)
- \( ∀ t ∈ T \setminus \{1\}, s_i^t = s_i^{t-1} + R_i^t - ∑_{j∈V} q_j^t \)

Phase 2 : Routing

Solved using metaheuristics (OR-Tools)
1 period = 1 CVRP problem such that :
- \( V = \{0\} \cup \{i | q_i^t > 0\} \)
- Special node 0 : supplier
- \( \{i | q_i^t > 0\} : \text{retailers} \)
- Demand at each node i : R_i = q_i^t

Results

<table>
<thead>
<tr>
<th>Instance</th>
<th>[1] Avg Time (s)</th>
<th>[2] Avg Time (s)</th>
<th>our approach</th>
</tr>
</thead>
<tbody>
<tr>
<td>S3PHC</td>
<td>399.45</td>
<td>200</td>
<td>0.02</td>
</tr>
<tr>
<td>S6PHC</td>
<td>188</td>
<td>119</td>
<td>0.02</td>
</tr>
<tr>
<td>B6PHC</td>
<td>119</td>
<td>119</td>
<td>1.30</td>
</tr>
</tbody>
</table>

References


Contact Information

- Web: https://fc.isima.fr/~axdelsol/
- Email: axel.delsol@isima.fr

axel.delsol@isima.fr