Routing problems with inventory and loading constraints

Axel Delsol

LIMOS, UCA

Objectives

- Study complex vehicle routing problems with inventory and loading constraints.
- Propose exact approaches : Mixed Integer Programming (MIP), Constraint Programming (CP) ...
- Propose heuristic approaches : metaheuristics, hybrid methods ...

First studied problem

- Multivehicle Inventory Routing Problem = Capacitated Vehicle Routing Problem + Inventory management
- Handle inventory level over time.
- Schedule transportation of products.

Capacitated Vehicle Routing Problem (CVRP)

Data :

1. Complete weighted graph : $G = (V, E)$
2. Weight of edge $(i, j) \in E$: $\mathcal{C}_{i,j}$
3. $V = \{0, \cdots, N\}$
4. Number of nodes : N
5. Special node $\mathbf{0}$: depot
6. Nodes $i \geq 1$: customers
7. Fleet of K vehicles with capacity $\mathcal{Q}$
8. Demand at each node $i \geq 1$: $\mathcal{R}_i$

Variables:

1. Vehicle assignment of node $i \geq 1$: v_i
2. Successor of node $i \geq 1$ in its route : x_i
3. First visited node of vehicle k : x_0^k

Objective: Minimize

Transportation cost : $\sum_{i \geq 1} \mathcal{C}_{i,x_i} + \sum_{k \in \mathcal{K}} \mathcal{C}_{0,x_0^k}$

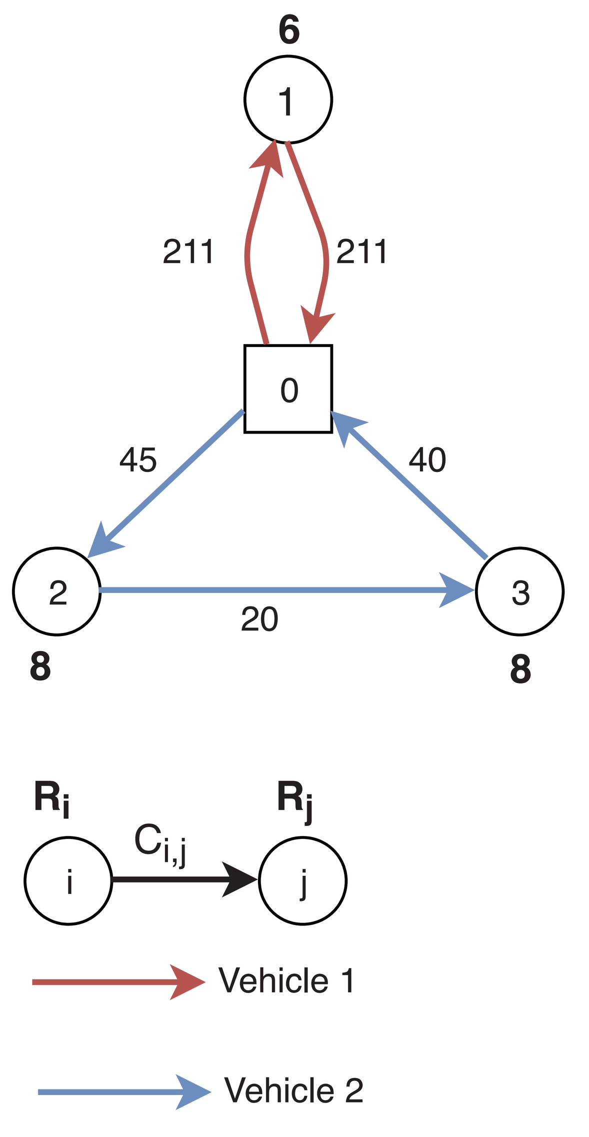

Figure 1: CVRP

Inventory management (IM)

Data :

1. Discrete time horizon : $\mathcal{T} = \{1, \cdots, T\}$
2. Demand at each node $i \geq 1$ (retailers) and period t : $\mathcal{R}_i^t$
3. Production at the supplier and period t : $\mathcal{R}_0^t$
4. Inventory bounds at node i : $[\mathcal{L}_i, \mathcal{U}_i]$
5. Inventory cost of node i at period t : $\mathcal{H}_i^t$
6. Initial inventory level at node t : $\mathcal{I}_i^0$

Variables:

1. Inventory level of node i at period t route : s_i^t
2. Delivery of node $i \geq 1$ at period t : q_i^t

Objective: Minimize

Inventory cost : $\sum_{t \in \mathcal{T}} \sum_{i=0}^{N} \mathcal{H}_i^t s_i^t$

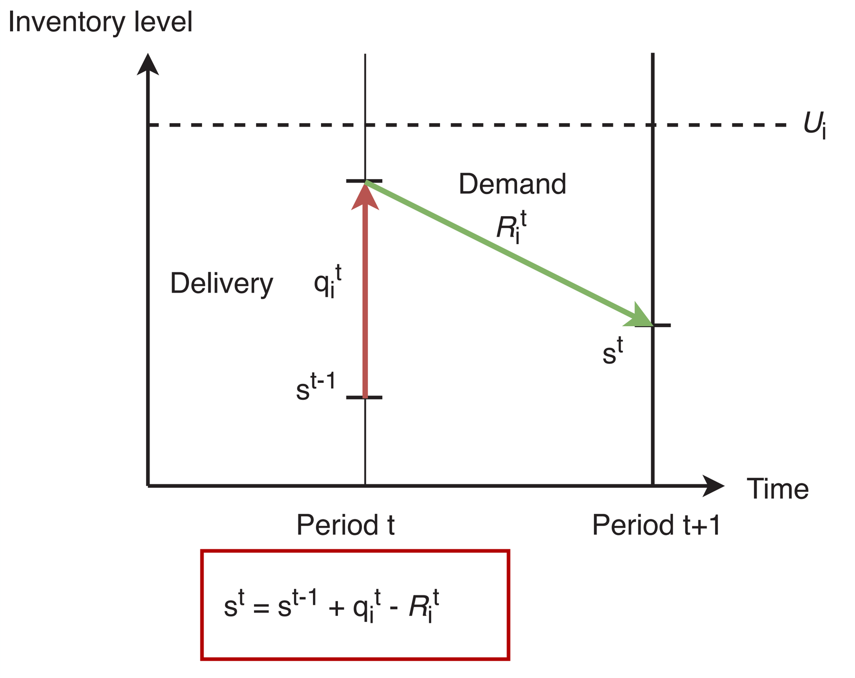

Figure 2: Inventory flow for one retailer

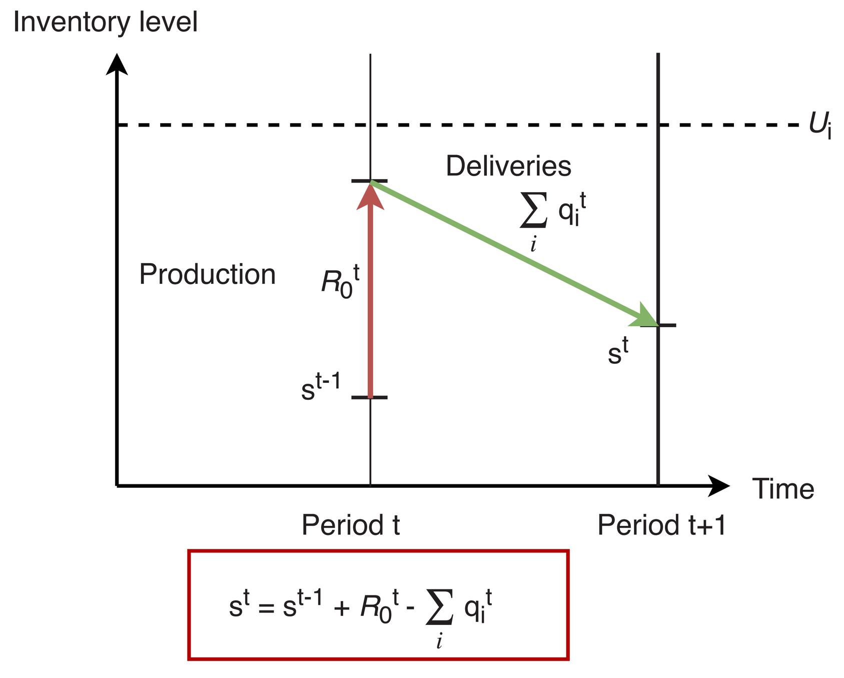

Figure 3: Inventory flow for the supplier

Contact Information

- Web: https://fc.isima.fr/~axdelsol/
- Email: axel.delsol@isima.fr

Resolution scheme

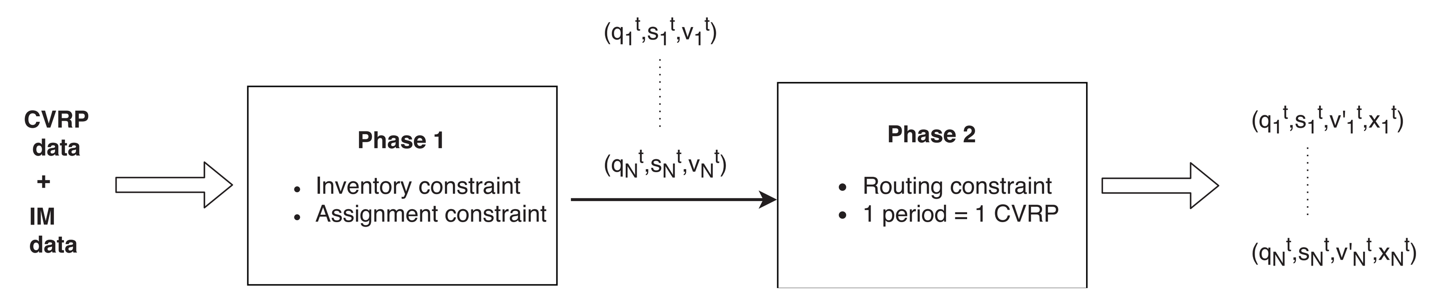

Figure 4: Global resolution scheme

Phase 1 : Feasible inventory solution

Solved using CP (OR-Tools : https://developers.google.com/optimization/)

Inventory constraints:

- $s_i^1 = \mathcal{I}_i^0 - \mathcal{R}_i^1 + q_i^1$
  $\forall t \in \mathcal{T} \setminus \{1\}, \quad s_i^t = s_i^{t-1} - \mathcal{R}_i^t + q_i^t$
- $s_0^1 = \mathcal{I}_0^0 + \mathcal{R}_0^1 - \sum_{i \in \mathcal{V}'} q_i^1$
  $\forall t \in \mathcal{T} \setminus \{1\}, \quad s_0^t = s_0^{t-1} + \mathcal{R}_i^t - \sum_{i \in \mathcal{V}'} q_i^t$

Assignment constraints:

- $\forall t \in \mathcal{T}, \quad \text{assignment}(\{v_i^t | i \in \mathcal{V}'\}, \{q_i^t | i \in \mathcal{V}'\}, K+1, \mathcal{Q})$

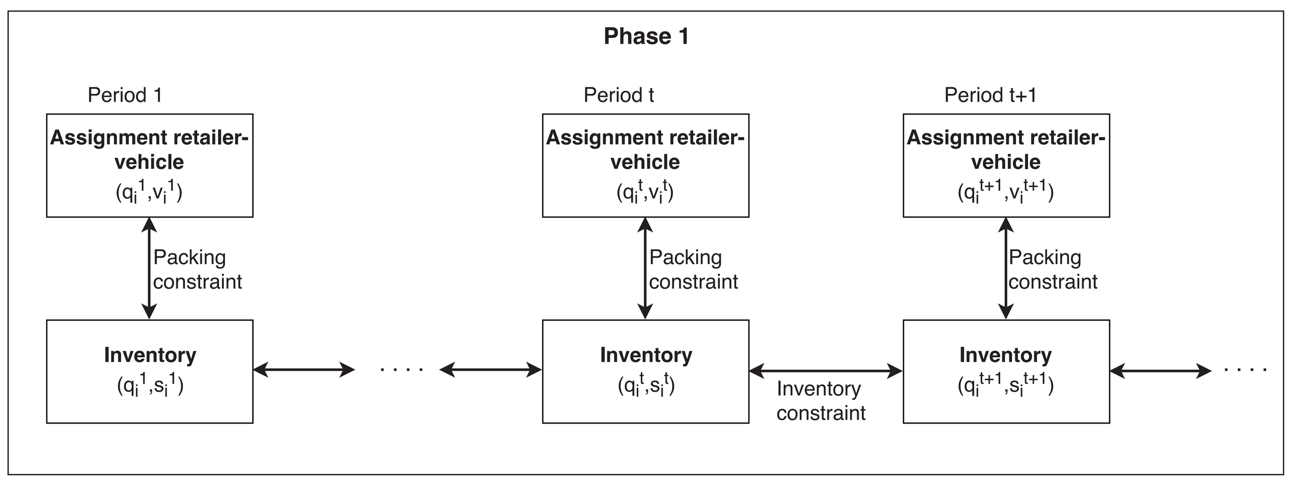

Figure 5: Phase 1 scheme

Phase 2 : Routing

Solved using metaheuristics (OR-Tools)

1 period = 1 CVRP problem such that :

1. $V = \{0\} \cup \{i | q_i^t > 0\}$
2. Special node $\mathbf{0}$: supplier
3. $\{i | q_i^t > 0\}$: retailers
4. Demand at each node i : $\mathcal{R}_i = q_i^t$

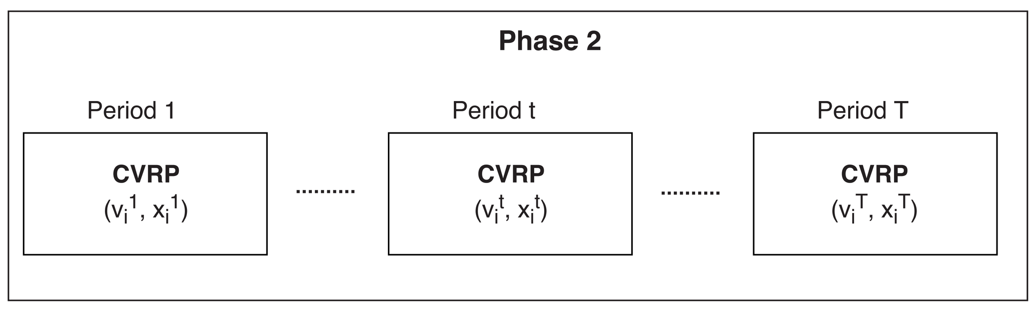

Figure 6: Phase 2 scheme

Results

Instance		[1]	[2]	our approach
S3PHC	Avg Time (s)	399.45	–	**0.02**
	Gap w.r.t best known solution	**6.48%**	36.8 %	17.25 %
	# sol	188	**200**	**200**
S6PHC	Avg Time (s)	504.30	–	**0.02**
	Gap w.r.t best known solution	**11.85** %	36.13 %	27.51 %
	# sol (s)	102	**119**	**119**
B6PHC	Avg Time (s)	–	–	**1.30**
	Gap w.r.t best known solution	–	31.76	**14.55**
	# sol (s)	–	**120**	120

References

[1] Claudia Archetti, Natashia Boland, and M. Grazia Speranza. A Matheuristic for the Multivehicle Inventory Routing Problem. *INFORMS Journal on Computing*, 29(3):377–387, aug 2017.

[2] Aldair Alvarez, Pedro Munari, and Reinaldo Morabito. Iterated local search and simulated annealing algorithms for the inventory routing problem. *International Transactions in Operational Research*, 25(6):1785–1809, nov 2018.

[3] Claudia Archetti, Luca Bertazzi, Alain Hertz, and M. Grazia Speranza. A Hybrid Heuristic for an Inventory Routing Problem. *INFORMS Journal on Computing*, 24(1):101–116, feb 2012.