# Optimization of Prestressed Concrete Bridges

UNIVERSITÉ Clermont Auvergne
Ecole doctorale Sciences Pour l'Ingénieur

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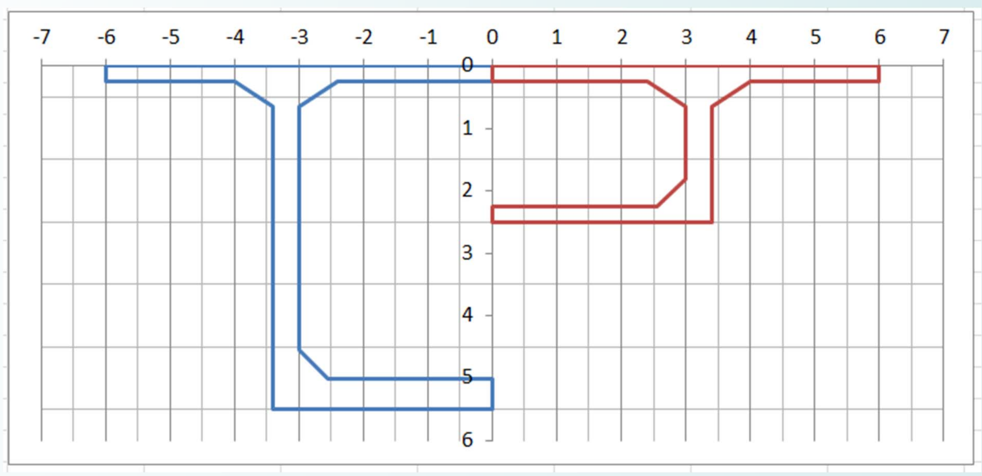
## Target

Optimization of prestressed concrete bridges aiming to minimize the cost while maintaining the bridge's functionality, has long occupied the mind of many researchers.

<u>**Objectives set for this study:**</u>

- Resolution of a multi-span beam with variable stiffness using a new developed analytical approach
- Introducing the new pivot rule in fully and partially prestressed concrete sections (SLS & ULS)
- Develop new methods of optimization for the design of prestressed concrete Bridges



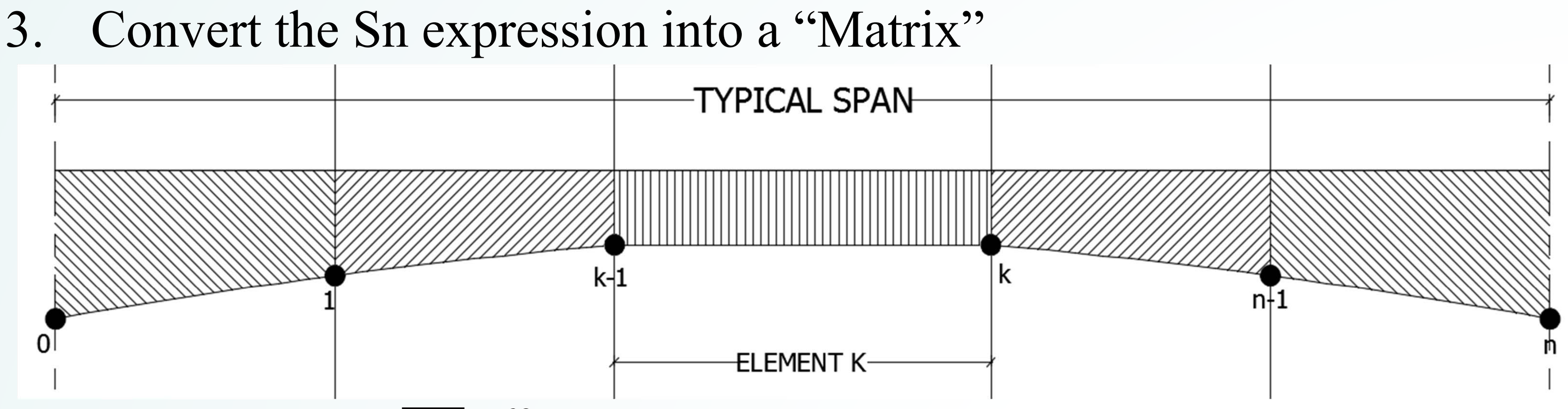
## Methodology

### New "Transfer Matrix" Method For Continuous Beams :

$$S_n = \int_0^l \frac{x^n}{l^n}\frac{dx}{EI}$$

<u>Main principles behind this Method</u>:

1. Use the integral Sn for the calculation of the Mechanical Constants (a, b & c) and the Rotations $\delta_i$ of the 3-Moment equation.
2. Choose the proper segmentation along the beam. ("Beam Meshing")
3. Convert the Sn expression into a "Matrix"




$$S_{k,n} = S_{k-1,n} + \sum_{j=0}^{n}\binom{n}{j}\beta_K{}^n\alpha_k{}^{n-j}(1-\alpha_k)^j \times I_{k,j}$$

$$\frac{x_k{}^n}{l^n} = \beta_k{}^n$$

$$\begin{bmatrix} S_{k,0}\\ S_{k,1}\\ S_{k,2}\\ S_{k,3}\end{bmatrix} = \begin{bmatrix} S_{k-1,0}\\ S_{k-1,1}\\ S_{k-1,2}\\ S_{k-2,3}\end{bmatrix} + A(\beta_k,\alpha_k)\begin{bmatrix} I_{k,0}\\ I_{k,1}\\ I_{k,2}\\ I_{k,3}\end{bmatrix}$$

$$\frac{x_{k-1}}{x_k} = \alpha_k$$

### New Pivot Rule for Fully and Partially Pre-stressed Concrete:

Main Equations to be satisfied by the EN:

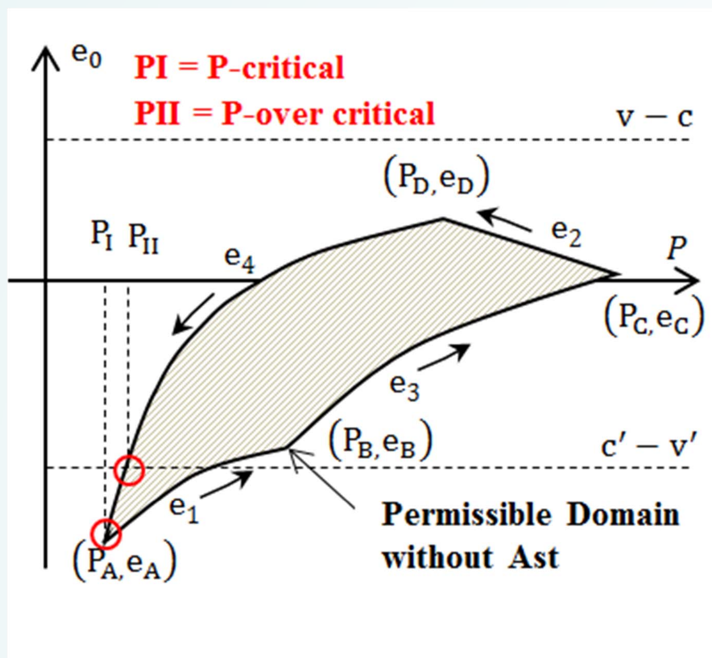
**Stress Limitation**

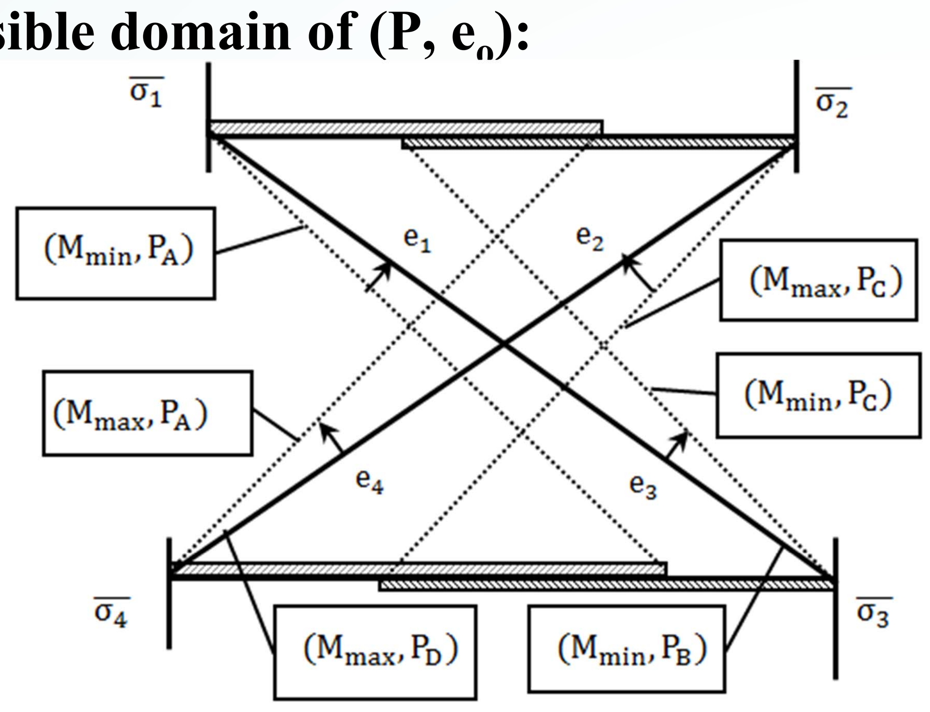
$$\overline{\sigma_1} \le \sigma_{sup}(M_{min}) \le \sigma_{sup}(M_{max}) \le \overline{\sigma_2}$$
$$\overline{\sigma_4} \le \sigma_{inf}(M_{max}) \le \sigma_{inf}(M_{min}) \le \overline{\sigma_3}$$

**Cover Limitation**

$$-(v'-c') \le e_0 \le v - c$$

**Permissible domain of (P, $e_0$):**



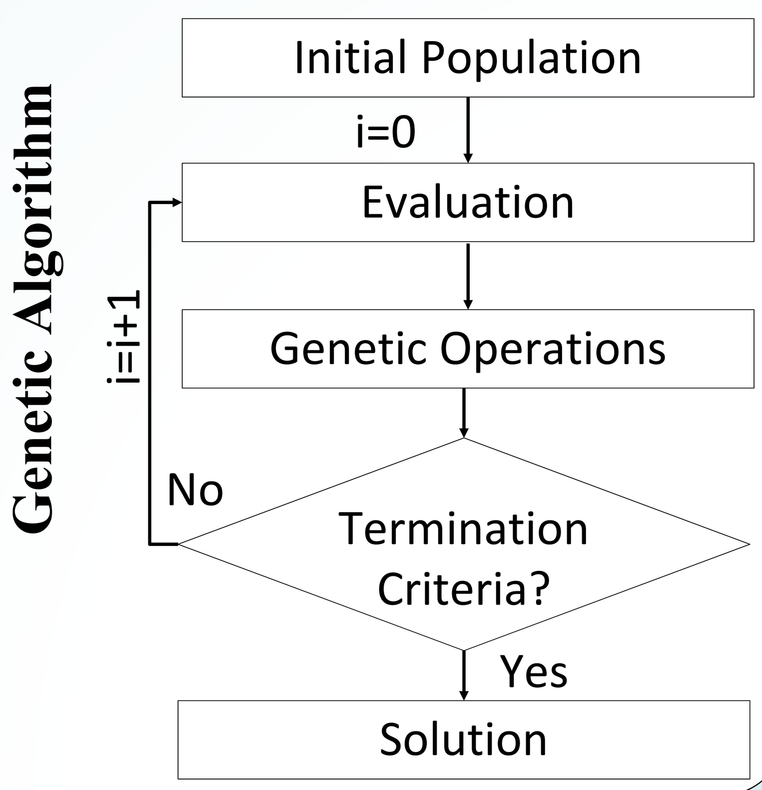

- Along each Hyperbola $e_i$ , Only <u>**1 constraint**</u> is satisfied
- At Intersection Points , <u>**2 constraints**</u> are satisfied <u>**simultaneously**</u>

### Optimization Method

Optimizing an objective function (Cost, P, Ac …) using either Analytical or Numerical Methods.

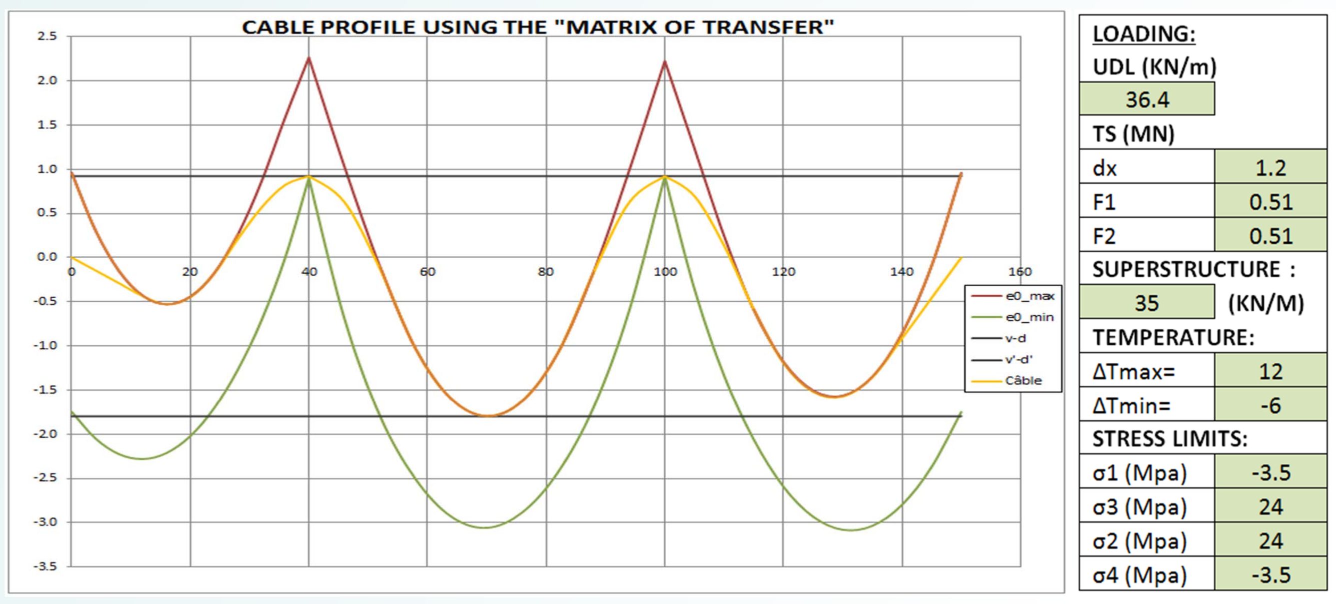
$$\begin{aligned} &\mathit{Minimize} && f(X)\\ &\mathit{Subject\ to} && g_j(X) < 0\\ & && h_k(X) = 0\\ & && x_{i,lower} < x_i < x_{i,upper}\end{aligned}$$

$$f: C_{conc}\,A_{conc} + C_{Ast}\,A_{Ast} + C_{Ap}\,A_{Ap} + \sum w_i\,Constraint_i$$




## Results

### Multi-Span PT beam using the "Matrix of Transfer:



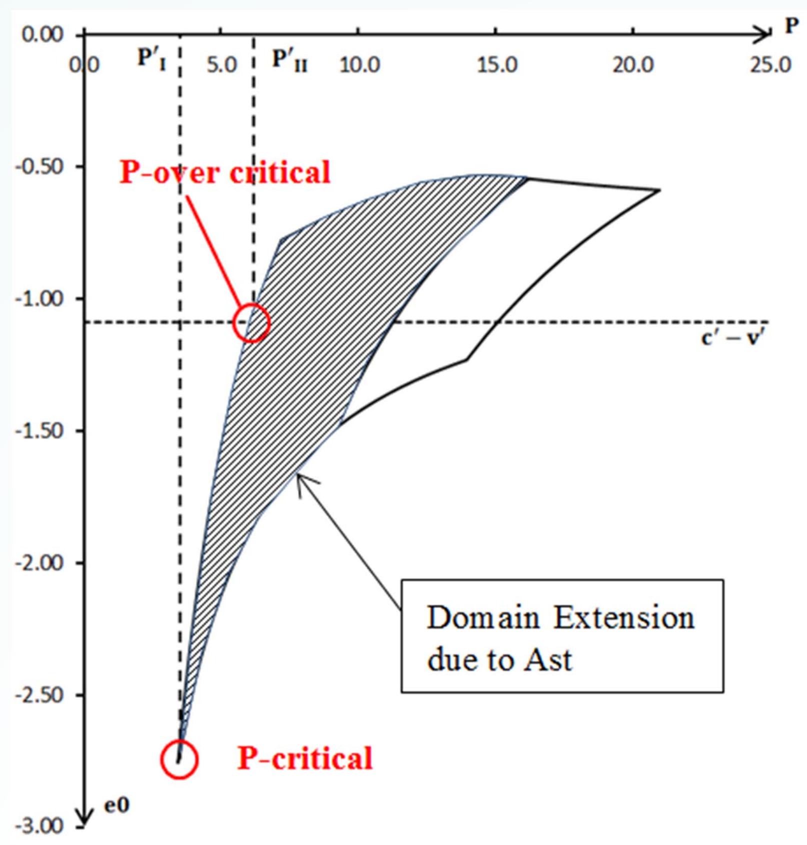

**Problem solved without the need of Numerical Methods!!**

$$\delta_i = \int_0^{l_{i+1}}\left(1-\frac{x}{l_{i+1}}\right)e0(x)\frac{dx}{EI} + \int_0^{l_i}\frac{x}{l_i}\frac{dx}{EI} = (S0-S1)_{i+1} + S1_i$$

← Rotation due to Pre-stressing moment formulated Using the "Matrix of Transfer"

### Optimization of Partial prestressing force P using Pivot Rule:

| Geometry | | | Stress Limits | | |
|---|---|---|---|---|---|
| b | 2 | m | $\overline{\sigma_1}$ | -2 | Mpa |
| h | 2.5 | m | $\overline{\sigma_2}$ | 24 | Mpa |
| $b_w$ | 0.24 | m | $\overline{\sigma_3}$ | 24 | Mpa |
| $h_f$ | 0.16 | m | $\overline{\sigma_4}$ | 0 | Mpa |
| $b_t$ | 0.8 | m | $W_k$ | 2 | mm |
| $h_t$ | 0.4 | m | **Material** | | |
| $h_g$ | 0.2 | m | $f_{ck}$ | 40 | MPa |
| $b_g$ | 0.4 | m | $f_{ct,eff}$ | 3.5 | MPa |
| $e_g$ | 0.2 | m | $E_s$ | 200 | GPa |
| **Cover** | | | $\alpha_e$ | 5.70 | |
| c | 0.2 | m | **Moments** | | |
| c' | 0.2 | m | $M_{min}$ | 2 | MN.m |
| $c_p$ | 0.04 | m | $M_{max}$ | 12 | MN.m |



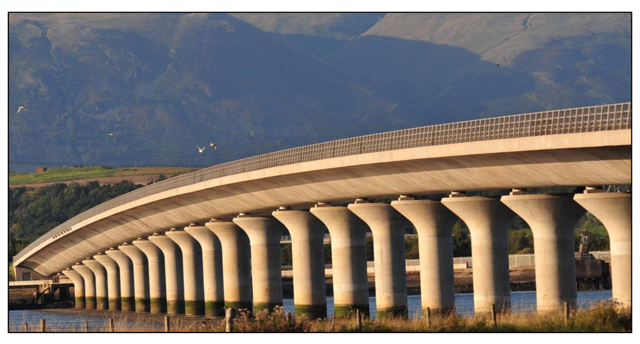

**Results under ELS-car:**

| | Case 1 | Case 2 | Case 3 | Case 4 |
|---|---|---|---|---|
| $A_{st}$(%) | 0 | 0.5 | 1 | 1.5 |
| $\overline{\sigma_{st}}$(Mpa) | 0 | 197 | 238 | 257 |
| P(MN) | 12.20 | 8.03 | 6.12 | 4.15 |

**Significant Reduction of P in Partially Prestressed Concrete = 60% for a 1.5% of Ast**

## Expected Results

- Find the portion of moment to be resisted by Ast for an optimal solution in Partial Prestressed Concrete
- Apply analytical optimization methods for 2 and 3 spans beams
- Develop numerical optimization tool, based on Genetic Algorithm, for a multi span prestressed beam with constant inertia and for a cantilevered bridge with variable inertia



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